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Computer Aided
Forest Road Designing

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1. INTRODUCTION

Nowadays we can witness extremely fast development of informatics, which the cheaper and cheaper microelectronics and the high-capacity computers make possible. This is why the computers are the essential aids of our daily lives. This is especially true by performance of the engineering tasks, where the ecological aspects get continually greater importance beside the technical and economic requirements. The synchronized consideration of these aspects can only be with drawing up and comparison of the many plan variants. We recognised this at our department and began a development of the road designing software. For the first time under DOS running program then until 2000 under widows working ERDÚTTERV program and until 2004 the maCADam software was finished.

According to our aims we wanted to create such a computer program, which from the viewpoint of didactics, can be fitted in the road designing process and at the same time helps the experts too, who work in the practice at the field of forest road design. To get rid of the many counting operations the construction engineer can spend more time to survey the whole plan.

The application of a computer is of great significance on two special fields of the forest road planning. One is the planning of opening up network the other is designing of single roads. In the first semester you can recognise the dynamic method of forest road planning with the help of the GIS software DigiTerra Map.

When designing the certain roads the most significant part of the constructor engineer work is the graphical planning of the road centreline in the work part of the general plan and of the profile in harmony with each other. On this the classical solution is the following: the road centreline is fitted together from series of tangents-curves-transition curves on the basis of the graphical plan.

According to the essence of another information technology tendency the road centreline is described with splines. Because of disadvantages this solution is worthy to use only when, if for example urban area in case of road modernization there are many side direction restrictions. Between these the spline can give a road centreline with smooth and continuously changing curvature.

2. HISTORICAL VIEW OF THE COMPUTER AIDED ROAD DESIGN

2.1. Developments in Europe

In Europe the application of computers for the planning of highways began in the year of 1960. In the beginning they needed computing centres for the running of the computer programs. Therefore these centres did the developments in co-operation with the planning offices.

In Hungary the Road-Railway Designing Enterprise (UVATERV) and the Department of Road Construction of the Technical University of Budapest performed the development of the UTESZ (Road designing with the help of computer) computer programs in the years of 1966-69. (Nemesdy, 1986)

In the same period in Switzerland the “WALD” special forest road designing software was developed by computing centres FIDES in co-operation with the profession department of the ETH Zürich (1977).

The speedy development of the microelectronics and computer science make the changing of the big computers to personal computers possible. So the computers could get on the desks of the engineers and an opportunity presented itself to make easier task-oriented programs. The high-capacity computers were replaced by personal computers. Mainly the IBM personal computers and for these written software spread. From these the German "STRATIS" (1990) road and civil engineering designing system can be underlined, which for the pre-planning and exact designing of the forest roads good usable. The French road designing program called “MicroPiste” was used in Hungary in the years of 1990. The Swiss “FOREST” and the Canadian “ROADENG” forest road designing software are also well useable, which have developed graphic possibilities.

The development of the high- and the mini-capacity computer systems can not be neglected, from among which the “MOOSS” (1985.) surface modeller system (on newer name GeoMACAO or MX) can be accounted civil engineer standard. This system was used for the plans of the “M3” motorway by the UVATERV.

Nowadays the development in leaps and bounds of the personal computers makes the application of the surface modeller and civil engineering establishment designing software possible. Concerning this a good example is the Autodesk Land Development Desktop and Civil Design designing software developed under AutoCad. In our days the development has been going on and useful software with more and more knowledge is appearing.

Also from the upper historical view the great development of the information technology and the application of that can be seen. Moreover it is observable that the advanced countries try set up their own software on this small area like the forest road design too.

2.2. Development at the University of West-Hungary

After these the question can be raised right away: why do we want to set up our own program if such software already exists The answer to this may be: as soon as we have possibility we have to obtain such a program. However we have to know in connection with foreign development the following difficulties can take place:

- all programs wish specific data survey and input, which is not suitable for home practice;
- the software making company not or just for significant sum makes available for us to input the translated text into the program;
- the prices of the really good and flexible software are very expensive.

We can make easier on the listed difficulties in a way that we shape cooperation with the software making company, we are supported by our own developments.

The first step of our University development was the application of the programmable calculator.(ifj. Gál, 1979)

The working up of the forest road design supporting program system written for the personal computer began in 1980 on the IBM 5110 PC, which was the first at our University. (Báthory, 1981) For lack of personal conditions the testing of the finished programs was interrupted and just in 1988 commenced again (EUTESZ). The steps of the development were the followings:

- the transcription of the already written programs,
- writing of the missing routines,
- fitting the subroutines into the program system,
- the visualization of the results in the AutoCAD.

The next step was the „ERDÚTTERV” software, which has its own graphic screen surface. At present the development of the new program (maCADam) is in process with the help of the created principles.

3. THE PLANNING PROCESS WITH THE HELP OF THE „EUTESZ” PROGRAM

At the planning of the opening up network we already decided the standard of the forest road, which we would like to design. After finishing the dimensioning of the pavement and choosing of slopes as a function of soil we can complete the cross-section template for straight road-sections, which is the first important input data of the design.

In the course of computer aided design we perform the traditional planning steps with convenient view order and content in accordance with the computer programs. Figure 3-1. shows the order of design-operations.

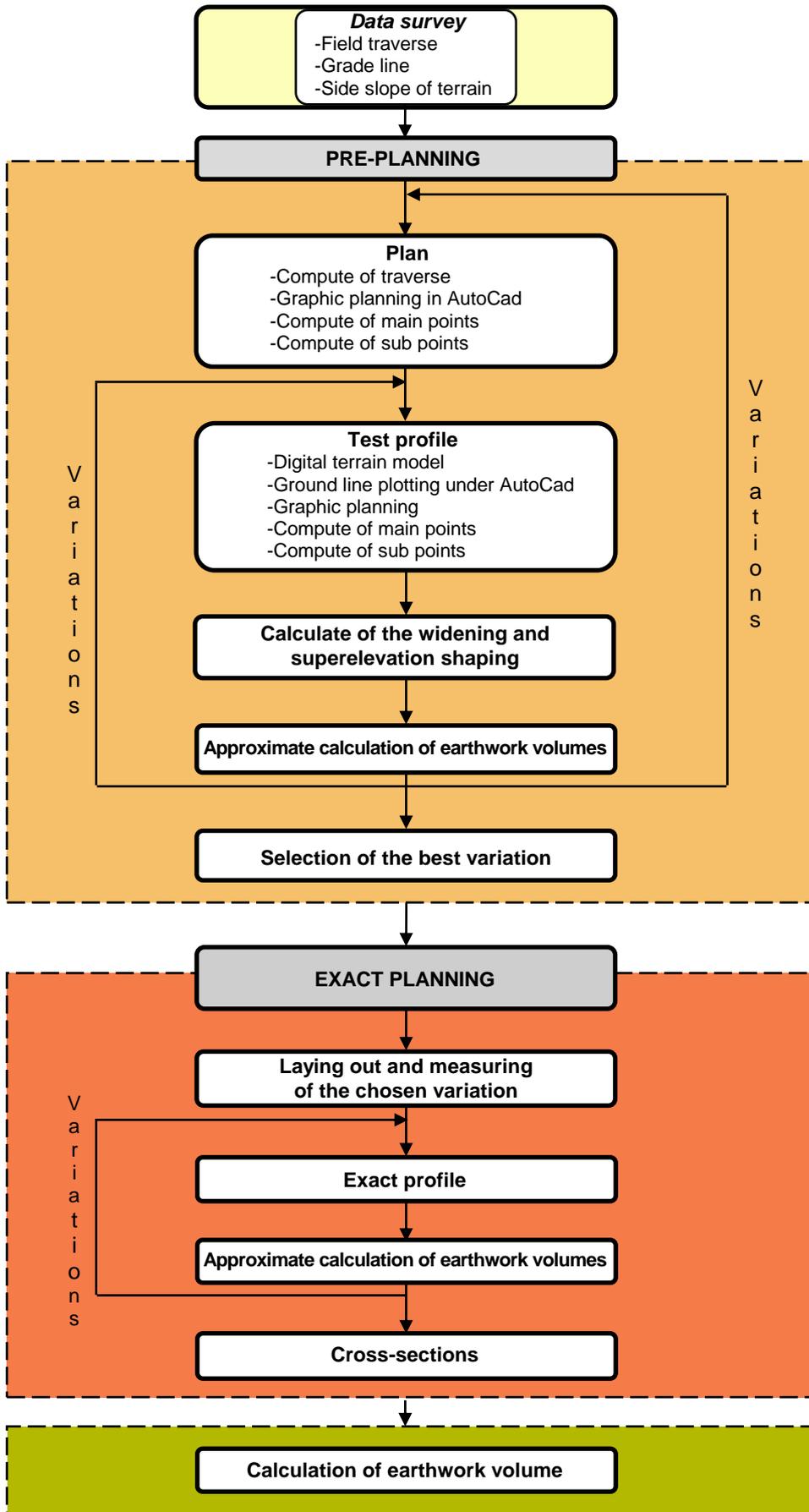


fig. 3-1.: Order of design-operations

3.1. Data Survey

The data survey starts with the laying out of the grade line on the mountain- and hill-country. Thereafter keeping the geodesic rules we stake out the field traverse of which the points of the grade line can be measured with the help of tangent offset method.

The input data of the calculating-designing program are the following:

- features of the grade line accompanying field traverse (traverse side length, left hand side embedded angle),
- the gradient between the points of the grade line,
- the relative coordinates of the grade line points referring to the traverse,
- side slope of the terrain measured at the points of the grade line.

The measuring of the terrain's side slope makes the establishment of a simple digital terrain model possible. With the help of this we can get the ground-lines of the profiles and cross-sections of the plan variations.

The designing occurs in two great steps which are the PRE-PLANNING and the EXACT PLANNING.

3.2. Pre-planning

The pre-planning serves that aim, that we can elaborate plan variations before we lay out the road centreline on the field, to choose from the most view points favourable solution.

3.2.1. Plan

According to the classical method the course of design we plot to the grade line well fitting tangents and curves, then we dissolve these to base situations, which the computer can calculate. If first the fitting of the tangents occurs, then this helps to realize a stretched dynamic horizontal alignment because by doing so we intersect a little waved sections of the grade line.

The planning of the horizontal alignment starts with the processing of the measured data. At first we get the coordinates of the points of the field traverse and of the grade line as well as on the basis of these graphic files. The graphic files with ".dxf" extension can be opened in AutoCAD where with the help of the drawing and editing commands we plot to the grade line tangents and curves, which we call main elements. Their location we give for the program by tangents with two points, by curves with two points and the radius. These input data can be queried in AutoCAD easily. The main elements are connected by spiral transition curves, which we call additional elements. Their data are

calculated by the computer partly or entirely. We call the tangential connecting points of the certain elements main points and the points between them sub points.

The road designing program executes the plan calculations in two steps. At first it calculates the coordinates of the main points of the plan and features of the curve elements then computes the coordinates of the sub points. The results of the calculations can be visualized in a graphical form if we open the created graphic files in AutoCAD. With this becomes possible the graphical verification too. To calculate the coordinates of the main points with the computer we dissolve the consecutive main elements to base situations. The base situations consist of the main element - additional element – main element triplet, which we give a one-digit number for the calculations. In the knowledge of the input data the program computes the additional elements or if between the main elements can not be placed additional element we get failure message.

Between the main points the coordinates of the sub points and of whichever point with given station value must be calculated. For this we have to give the distance between the sub points in the straight road section further depending on the radius in curve and the station value of the special points, which we want to lay out. If we give, however from which traverse side and which station value we want to segment the centreline, we get the orthogonal and polar laying out data of main and sub points referring to field traverse.

After the overlooking of the location of the designed plan centreline, with the modification of the input data of elements of incorrectly fitting parts, the horizontal alignment can be redesigned easily.

3.2.2. Test Profile

The main purpose of the profile is the graphical designing of the vertical alignment and the elevations of the points of the plan centreline.

3.2.2.1. Digital Terrain Model

Before laying out with the help of the digital terrain model maker subroutine the elevations of the points of the plan centreline can be determined. The program uses a quadrangle net model. The points of quadrangle are the following:

- two consecutive grade line points with three coordinates,
- in accordance with going direction from the two grade line points 10,00 metre on the right two points with three coordinates.

To the generation of these points we survey the input data by the mentioned measuring of the grade line.

3.2.2.2. Profile

We perform the determination of the altitude-data in the pre-planning phase with the help of the digital terrain model. As a result of the calculation we get the ground line of the profile of the plan variation. After the plotting of the ground line of the profile with plotter, the test profile can be completed.

After the graphical planning of the vertical alignment, we determine the elevations of the points of the road centreline. The program determines first the main points' then the sub points' elevations, alike the plan designing. In the profile the main elements are the tangents (sustained grades), between which the computer calculates the vertical curves as additional elements. We name vertical main points the breaks of the designed vertical road centreline further the beginning- and the endpoints of the vertical curves. We give the tangents with station value and elevation of their one point and with their grade. We characterize the vertical curves according to grade changing method with the length of the tangents and with the grade-changing value.

As a result of the program run we get the station values and the elevations of the beginning- and the endpoints of the tangents further of the tangential points of vertical curves. If we ask for the forepart of the program run, we get the graphical profile in the form of graphic file too. Opening this file in AutoCAD we can verify the vertical alignment. The computer finishes the newer calculation with the occasional modification quickly and accurately. After the running of the subroutine of the sub point computing we get the elevations of all main and sub points, which means the subgrade or equalizing subgrade level.

3.2.3. Calculation of the Widening and Superelevation Shaping

As input data just the width and in the straight road section applied side slope of the surface must be given. The other data are already available on the basis of previous calculations.

Because this subroutine makes verifications during the running, it is suitable to run after using of the terrain model before the doing of the test profile. As a result of the program run we get the widening and superelevation values of the centreline points, which are in the shaping section and in the curves. With the help of these we can make the approximate calculation of earthwork volume more correct.

3.2.4. Approximate Calculation of Earthwork Volumes

To the approximate calculation of earthwork volume we need the subgrade width, the angles of the slopes, the modified ditch depth and the thickness of the humus layer. The side slope of the ground line can be determined with the help of the terrain model. To the determination of the areas of the cut and fill of the cross-sections necessary other data are available as a result of previous program runs. The computer calculates the approximate earthwork volume with the help of the areas of cuts and fills and with the distance between the cross-sections. Apart from calculation of areas and earthwork volume we have possibility to visualize the cross-sections on the display. The drawings of the cross-sections can be printed too.

3.2.5. Selection of the Best Variation

With the changing of the input data of the above subroutines we can work out variations. As in the flowchart (fig. 3-1.) it can be seen, we can change the horizontal and the vertical alignment, respectively the cross-sections. After these we can compare and consider the solutions then we can choose the most favourable for us. The viewpoints of the comparison are the following:

- the calculation of the earth volume,
- occupied area by road prism
- the necessity foundation of fill,
- the necessary structures (bridges, culverts),
- other viewpoints.

3.3. Exact Planning

We select the most favourable plan centreline from the variations and lay out in the field with the help of the marking out data, which the computer calculated. At the same time we measure the additional field points, which are necessary to the designing. After this we determine the elevations of the laid out centreline points with levelling and survey the ground lines of the cross-sections.

As a next step of the exact planning we have to run the program again, which calculates the widening and superelevation values, because we were allowed to insert or to leave sub points.

The program makes the graphic file of the exact ground profile from the elevations of the laid out centreline points, which we determine with levelling. To the design of the vertical alignment we can use up the profile and the

approximate earthwork volume calculating subroutines again, which was introduced by the pre-planning. To the calculations of approximate earthwork volume we have to measure and give the average side slope of the exact ground lines of the cross-sections, on the basis of field surveying of the ground lines of the cross-sections.

Because the tangents of profile can be changed easily and the approximate earthwork volume can be calculated immediately, the most favourable vertical alignment can be designed relatively in a short time.

To the lifting of the tangents on the road surface level in the profile we have to modify the elevation of the previously specified tangents' points with the elevation difference between the subgrade and surface level.

The program system aids the further operations of the final variation no more (Detailed Structure Designs, Technical description, Calculation etc.), so the performance of those is the equivalent to the traditional design.

4. APPLIED PROCEDURES AND RUNNING OF THE COMPUTER PROGRAMS

In this chapter we introduce the basic principles of the programs, and the applied procedures.

4.1. Data Survey and Processing

The coordinate system used to the plan designing is a clockwise rotation system, which we use in general in geodesy, namely the "X" axis with right direction rotation gets in "Y" respectively the elevation meaning in "Z".

We need geodesic reference point network to the determination of coordinates belonging to the breakpoints of the grade line and the terrain as well as to the laying out of the road centreline. In the case of road design a field traverse provides this task. If everything turns out well, this traverse is at the same time itself the centreline traverse but this may be a general field traverse too. If we perform the measuring of field traverse with theodolite and band-chain, we calculate the points of traverse with the method which we had recognized within the frame of the subject of geodesy. If we use a total station for the measuring, after downloading the data into the computer, the geodesic calculation and the generation of the surface model (TIN) can be performed.

4.2. Main Point Calculation in Plan

The task of the main point calculation in plan:

- the determination of the main points of the centreline, which means in case spiral transition curve beginning of transition curve “BTC” and end of transition curve “ETC”, in case circular curve beginning of curve “BC” and end of curve “EC”.
- the verification of the connection of main and additional elements.

4. 2.1. The Principle of the Main Point Calculation

In the plan we distinguish main and additional elements. We name main elements the circular curves and tangents (the straight sections), the location of which we predetermine. The main elements connect with additional elements (spiral transition curve, perhaps circular curve) to one another. We don't or just partly define the additional elements because these can be calculated with the help of convenient formulas and algorithm. The main element – additional element – main element triplets always constitute base situations, which we give with one-digit number. Knowing these the program computes the additional elements or if between the main elements can not be placed additional element we get failure message. If the coordinates of the main points can be determined, then the sub points always can be calculated too.

4.2.1.1. Specification of the Main Elements

4.2.1.1.1. Specification of the Tangent

We always give tangents with pair of coordinates of their two points in metres with two decimal accuracy. We have to pay attention to the order of the points, which must follow one another always according to the direction of the placing stations. Namely we can get from the first point to the second point if we move forward on the straight road section in direction of the placing stations.

4.2.1.1.2. Specification of the Circular Curve

Two cases are possible by the specification of the circular curve:

a.) As a main element: with its two points and radius

We give the two points according to the tangents. At the same time we have to give the radius of the circular curve like sign rule. The explanation of this:

- the radius is positive if the curve has a right direction,
- the radius is negative if the curve has a left direction

We give the value of the radius and the coordinates of the two curve points both with centimetre accuracy. Because of the agreement on the circular curve the series of the points we explain so, that from by the first and second point defined chord the centre point of the curve locates to the right if the curve has a right direction, to the left if the curve has a left direction.

b.) As an additional element: only with its radius.

The geometric location of the circular curves can be calculated between two specified tangents. If we give the parameter of the transition curve, the circular curve with spiral transition curves can be calculated too.

4.2.1.2. Used procedures

To the calculation of the main points we use the following formulas and procedures: the distance of two points "T", which are given with "X, Y" absolute coordinates:

$$T = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2} . \tag{3.5}$$

- the absolute coordinates of the main element "X, Y", which is given by the relative coordinates "η, ξ" referred to the traverse side:

$$\begin{aligned} X &= X_i + \eta \cdot \cos\delta - \xi \cdot \sin\delta, \\ Y &= Y_i + \eta \cdot \sin\delta + \xi \cdot \cos\delta, \end{aligned} \tag{3.6}$$

in which:

- X_i, Y_i : coordinates of the one endpoint of the traverse side;
- δ : azimuth of the traverse side, calculated with $\delta = \arctg \frac{Y_{i+1} - Y_i}{X_{i+1} - X_i}$.

The coordinates of the point in the new coordinate system (relative coordinates referred to the traverse side or a tangent) "η, ξ":

$$\begin{aligned} \eta &= (X - U) \cdot \cos\delta + (Y - V) \cdot \sin\delta, \\ \xi &= (Y - V) \cdot \cos\delta - (X - U) \cdot \sin\delta, \end{aligned} \tag{3.7}$$

where:

- U, V: the coordinates of the starting point of the new coordinate system;

- δ : the rotation angle of the new system of coordinate (the azimuth of the traverse side or the tangent determined with $\delta = \text{arctg} \frac{Y_{i+1} - Y_i}{X_{i+1} - X_i}$

formula);

- X, Y: the coordinates of a point in the original coordinate system.

To determine the features of the spiral transition curve the following formulas serve :

$$p = \sqrt{R \cdot L} \quad \text{or} \quad L = \frac{p^2}{R}, \quad (3.8)$$

$$x = l - \frac{l^5}{40p^4} + \frac{l^9}{3456p^8}, \quad y = \frac{l^3}{6p^2} - \frac{l^7}{336p^6} + \frac{l^{11}}{42240p^{10}}, \quad (3.9)$$

$$\Delta R = \frac{L^2}{24R} - \frac{L^4}{(24R)^2} \cdot \frac{1}{4,67R}, \quad (3.10)$$

- the "L" is expressible, after the formula (3.10) is reduced to the second degree:

$$L = \left[\frac{24 \cdot 4,67R^2 (1 - (1 - (4\Delta R/4,67R))^{1/2})}{2} \right]^{1/2}, \quad (3.11)$$

$$X_0 = \frac{L}{2} - \frac{L^3}{240R^2}, \quad (3.12)$$

where the explanation of the certain marks are the following:

- p: the parameter of the transition curve;
- L: the length of the transition curve;
- R: the radius of the circular curve;
- x, y: the relative coordinates of the points of the transition curve referred to the main tangent and calculated from the beginning of the transition curve;

At first we determine the azimuth " δ " of the tangent with the formula of $\delta = \text{arctg} \frac{Y_{i+1} - Y_i}{X_{i+1} - X_i}$, then the distance "TH" of two points of the circular curve with the (3.5) and their relative coordinates " η, ξ " referred to the tangent with the (3.7) formulas. The starting point of the coordinate system of " η, ξ " is the point "A".

The next step is the calculating of the angle " β " from the "CDE" triangle.

$$\beta = \text{arctg} \frac{\text{TH}}{\sqrt{4R^2 - \text{TH}^2}}. \quad (3.13)$$

After this we calculate the azimuth " δ' " of the tangent, which passes through the points "C, D" in the coordinate system " η, ξ " given by points "A, B" with the formula $\delta' = \text{arctg} \frac{\xi_2 - \xi_1}{\eta_2 - \eta_1}$. Thereafter the circular curve's offset of the transition curve " ΔR " can be calculated:

$$\Delta R = \xi_2 - R(1 - \cos(\delta' + \beta)). \quad (3.14)$$

After we have calculated the length of the transition curve "L" as well as the abscissa value of the circular curve's centre " X_0 " with the formulas of (3.11) and (3.12), the distance is between the beginning point of the transition curve and the point "A":

$$\text{TK} = \eta_2 - R \cdot \sin(\delta' + \beta) - X_0. \quad (3.15)$$

We can get the coordinates "X, Y" of the beginning point of the transition curve with the formula (3.6), in which:

- $\eta = \text{TK}$;
- $\xi = 0$;
- $X_i = X_A$;
- $Y_i = Y_A$;
- δ : azimuth of the tangent, which we have determined in the first step.

We can determine the relative coordinates "x, y" of the end of transition curve referred to the main tangent "A, B" and calculated from the beginning of transition curve with the formula (3.9). At the substitution:

- $l = L$;
- p : the parameter of transition curve, which we get from the formula (3.8).

We get the coordinates "X, Y" of the point "ETC" also the using up of the formula (3.6), in which the explanation of the certain variable is the following:

- $\eta = TK + x$;
- $\xi = y$;
- the value of the other variables agrees with the written things of the point "BTC".

The 2nd base situation is the mirror image of the 1st base situation, therefore the now introduced course of calculation here is applicable too, after we have inverted the order of the specified points on the tangent and changed the sign of the radius to the contrary of the earlier.

4.2.1.3.2. The Principle of the Calculation of the 3rd and 4th Base Situations

At the base situations of 3rd and 4th our task is the determination of the coordinates "X, Y" of the main points of the circular curve or circular curve with spiral transition curves (BC, EC, BTC, ETC), which connect with two tangents tangential. For this the following data are available (fig. 4.2.1.3.2.-1.):

- the coordinates of two points of the first tangent, which is tangential to the circular curve or circular curve with spiral transition curves:
 - A: X_A és Y_A ,
 - B: X_B és Y_B ,
- the coordinates of two point of the second tangent, which is tangential to the circular curve or circular curve with spiral transition curves:
 - C: X_C és Y_C ,
 - D: X_D és Y_D ,
- the radius of circular curve with its sign;
- at the base situation 4th, the parameters of the entering and exiting transition curves (p_1, p_2).

After determination of the azimuths (δ_1, δ_2) of the two tangents with the formula: $\delta = \text{arctg} \frac{Y_{i+1} - Y_i}{X_{i+1} - X_i}$, we get the coordinates of their intersection point (X_M, Y_M) as follows:

$$\begin{aligned} Y_A &= X_A \cdot \text{tg}\delta_1 + B_1, \text{ from which } B_1 = Y_A - X_A \cdot \text{tg}\delta_1 \\ Y_C &= X_C \cdot \text{tg}\delta_2 + B_2, \text{ from which } B_2 = Y_C - X_C \cdot \text{tg}\delta_2 \end{aligned} \tag{3.16}$$

$$Y_M = X_M \cdot \text{tg}\delta_1 + B_1 = X_M \cdot \text{tg}\delta_2 + B_2,$$

$$X_M = \frac{B_2 - B_1}{\text{tg}\delta_1 - \text{tg}\delta_2}.$$

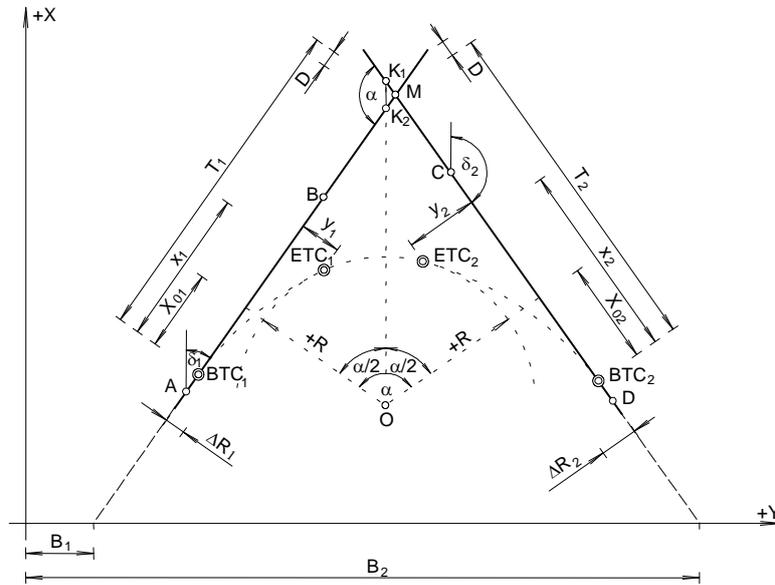


fig. 4.2.1.3.2.-1.:
Geometric meaning of the 3rd and 4th base situations

Included angle of the two tangents " β ":

(3.17)

$$\begin{aligned} \beta_1 &= \delta_2 - \delta_1, \\ \text{if } \beta_1 < 0^\circ, & \text{ then } \beta = \beta_1 + 360^\circ, \\ \beta_1 > 180^\circ, & \text{ then } \beta = 360^\circ - \beta_1, \\ & \text{else } \beta = \beta_1 \end{aligned}$$

After this the central angle of the curve " α ":

- if $\alpha < 180^\circ$, then

$$\alpha = \beta$$

- if $\alpha > 180^\circ$

$$\alpha' = 360^\circ - \beta$$

Thereafter we need the length " L ", the circular curve's offset " ΔR " and the abscissa value of the circular curve's centre " X_0 " of transition curves to calculation of the relative coordinates of the main points referring to tangents. We can get these values by the aid of the input data (radius and parameters) with the formulas (3.8), (3.10) and (3.12).

Depending on the central angle " α " the used formulas are the following:

- if $\alpha < 180^\circ$

(3.18)

$$D = \frac{\Delta R_1 - \Delta R_2}{\sin \alpha}$$

(3.19)

$$T_1 = X_{01} + (R + \Delta R_1) \cdot \operatorname{tg} \frac{\alpha}{2} - D$$

$$T_2 = X_{02} + (R + \Delta R_2) \cdot \operatorname{tg} \frac{\alpha}{2} + D$$

- if $\alpha > 180^\circ$

(3.20)

$$D_N = \frac{\Delta R_1 - \Delta R_2}{\sin \alpha'}$$

(3.21)

$$T_1 = (R + \Delta R_1) \cdot \operatorname{tg} \frac{\alpha'}{2} - X_{01} - D_N$$

$$T_2 = (R + \Delta R_2) \cdot \operatorname{tg} \frac{\alpha'}{2} - X_{02} + D_N$$

The explanation of marks used in the above mentioned formulas are the following:

- D and D_N : the projections in the direction of tangents of the deviation, which arising from several circular curves' offsets of transition curves (distance between the points " K_1, M " and " K_2, M " in figure 4.2.1.3.2.-1.).
- T_1, T_2 : distances (lengths of tangents) from the point of intersection of the tangents to the beginnings of transition curves. In case of circular curve without transition curves " T_1 " = " T_2 ", because " X_0 ", " ΔR " and " D " are zero.

We can get the relative coordinates of the ends of transition curves referred to tangents, calculated from the beginnings of transition curves if we substitute in the formula (3.9), in which:

- $l = L_1$, then $l = L_2$;
- $p = p_1$, then $p = p_2$.

Ultimately we determine the coordinates of the main points " X, Y " with the formula (3.6), in which:

- $X_i = X_M$;
- $Y_i = Y_M$;

if $\alpha < 180^\circ$

- In case of points BTC_1, BTC_2 ; BC, EC :

$$\eta = -T_1, \text{ respectively } \eta = +T_2;$$

$$\xi = 0;$$

- In case of points ETC₁, ETC₂:

$$\eta = x_1 - T_1, \text{ respectively } \eta = T_2 - x_2;$$

$$\xi = \pm y_1, \text{ respectively } \xi = \pm y_2,$$

where the signs are shaped by the sign of radius;
if $\alpha > 180^\circ$

- In case of points BTC₁, BTC₂ ; BC, EC:

$$\eta = +T_1, \text{ respectively } \eta = -T_2;$$

$$\xi = 0;$$

- In case of points ETC₁, ETC₂:

$$\eta = x_1 + T_1, \text{ respectively } \eta = -T_2 - x_2;$$

$$\xi = \pm y_1, \text{ respectively } \xi = \pm y_2,$$

where the signs are shaped by the sign of radius.

4.2.1.3.3. The Principle of the Calculation of the 5th and 6th Base Situations

At the base situation 5th, the location of the transition curve – tangent – transition curve additional elements must be determined, which are between two given circular curves. The input data are the following:

- the coordinates of two points of the first circular curve:

$$A: X_A \text{ és } Y_A,$$

$$B: X_B \text{ és } Y_B;$$

- the radius of the first circular curve "R₁" with sign;

- the parameter of exiting transition curve of the first circular curve "p₁";

- the coordinates of two points of the second circular curve:

$$C: X_C \text{ és } Y_C,$$

$$D: X_D \text{ és } Y_D;$$

- the radius of the second circular curve "R₂" with sign;

- the parameter of entering transition curve of the second circular curve "p₂";

- the azimuths of chords of the circular curves " δ_1, δ_2 " are available with the formula $\delta = \arctg \frac{Y_{i+1} - Y_i}{X_{i+1} - X_i}$ if we substitute of the coordinates of points of the circular curves "A, B, C, D".
- we can get the coordinates of the centre points (U_1, V_1 and U_2, V_2) with the help of the formula (3.6), in which:
 - $X = U_1$, illetve $X = U_2$;
 - $Y = V_1$ respectively $Y = V_2$;
 - $X_i = X_A$, respectively $X_i = X_C$;
 - $Y_i = Y_A$, respectively $Y_i = Y_C$;
 - $\eta = T_1$, respectively $\eta = T_2$;
 - $\xi = K_1$, respectively $\xi = K_2$;
 - $\delta = \delta_1$, respectively $\delta = \delta_2$.

After this the azimuth " δ " of the two centre points " O_1, O_2 " connecting straight line can be calculated with substitution the coordinates of " U_1, V_1 " and

" U_2, V_2 " in the formula $\delta = \arctg \frac{Y_{i+1} - Y_i}{X_{i+1} - X_i}$.

Thereafter we determine the azimuth of the tangent of the transition curves as well as the common point of this tangent and the " O_1, O_2 " centres connecting straight line. The result can be achieved in different ways depending on whether we design the curves with the same direction or reversed.

In Case of Curves with Same Direction:

- The perpendicular distances between the centres " O_1, O_2 " and the found tangent (R_3, R_4), which are equal with length of the sections " O_1, P " and " O_2, Q ", on the basis of the radiuses and of the circular curve's offsets " $\Delta R_1, \Delta R_2$ " calculated with the formula (3.10) are the following (Fig. 4.2.1.3.3-1.):

$$R_3 = R_1 + \Delta R_1, R_4 = R_2 + \Delta R_2, \tag{3.24}$$

where we calculate with the absolute values of the radiuses (4.2.1.3.3-1. ábra);

- We can state from the analogies of triangles "O₂, M, Q" and "O₁, M, P" the following:

(3.25)

$$\frac{T+x}{R_4} = \frac{x}{R_3}, \text{ then } x = \frac{T \cdot R_3}{R_4 - R_3},$$

where:

- T: distance between the centres "O₁, O₂";
- x: distance between the centre of the first circular curve "O₁" and the wanted point of intersection "M";
- We get the wanted coordinates "X_M, Y_M" with the substitution in the formula (3.6):
 - X = X_M
 - Y = Y_M
 - X_i = U₁;
 - Y_i = V₁;
 - η = -x;
 - ξ = zero;
 - δ = δ'.

- the azimuth of the wanted tangent "δ_E":

(3.26)

$$\beta = \arctg \frac{R_3 - R_4}{\sqrt{T^2 - (R_3 - R_4)^2}} \cdot (\text{sign of } R_1),$$

$$\delta_E = \delta' + \beta.$$

In Case of Reverse Curves:

- we can state from the analogies of the triangles "O₁, M, P" and "O₁, O₂, S" as well as "O₁, M, Z" and "O₁, O₂, H" the following (Fig. 4.2.1.3.3-2.):

(3.27)

$$\frac{R_3}{R_3 + R_4} = \frac{t}{T}; \quad \frac{t}{T} = \frac{x}{U_2 - U_1}, \text{ from where } x = \frac{R_3(U_2 - U_1)}{R_3 + R_4},$$

$$\text{then on the basis of these } y = \frac{R_3(V_2 - V_1)}{R_3 + R_4};$$

- the coordinates "X_M, Y_M" of the point of intersection "M" with the help of the above formulas for "x" and "y":

(3.28)

$$X_M = x + U_1, \quad Y_M = y + V_1,$$

(3.29)

$$X_M = \frac{R_3 U_2 + R_4 U_1}{R_3 + R_4}, \quad Y_M = \frac{R_3 V_2 + R_4 V_1}{R_3 + R_4};$$

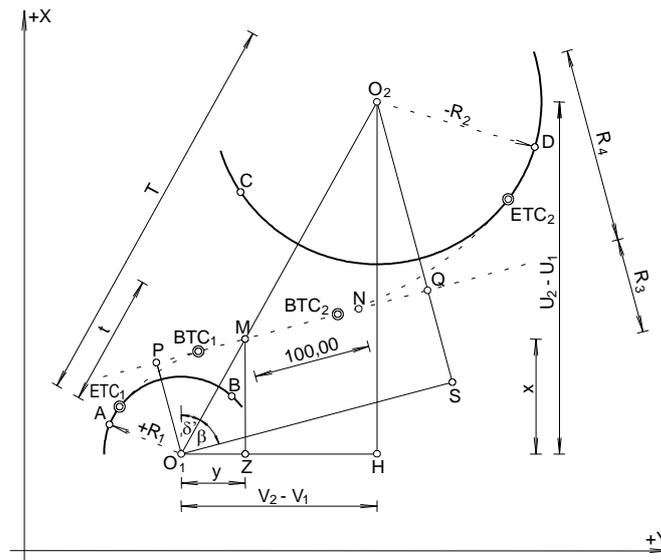


fig. 4.2.1.3.3.-2.:

Geometric meaning of the 5th base situations in case of reverse curves

- the azimuth of the wanted tangent "δ_E":

(3.30)

$$\beta = \arctg \frac{R_3 + R_4}{\sqrt{T^2 - (R_3 + R_4)^2}} \cdot (\text{sign of } R_1),$$

$$\delta_E = \delta' + \beta.$$

We can take down the point "N", which is on the tangent, optional distance from the point of intersection "M". The program calculates with 100.00 metres, so the wanted coordinates " X_N, Y_N " can be achieved with the formula (3.6). By substitution the variables mean the following:

- $X = X_N$
- $Y = Y_N$
- $X_i = X_M$;
- $Y_i = Y_M$;
- $\eta = 100,00$;
- $\xi = \text{zero}$;
- $\delta = \delta_E$.

After this the 2nd then the 1st base situation can follow. With the help of these coordinates of the main points " $ETC_1, BTC_1, BTC_2, ETC_2$ " of the transition curves can be determined. The program calculates at first the main points of the exiting transition curve of the first circular curve, then the main points of the entering transition curve of the second circular curve.

The 6th base situation uses the 5th base situation so many times, as much as needs for the gradual reduction to zero of the straight section, which is between the beginning points of the transition curves " BTC_1, BTC_2 ".

4.2.1.3.4. The Principle of the Calculation of the 7th base situation

The 7th base situation deviates from the previous base situations. The purpose of this is the elimination of the short straight section between the curves, which have the same directions. This way we can shape a compound curve, which is reachable with the increasing of radius of the circular curve, therefore at least one has to be circular curve. We calculate the new radius " R' " with the help of the new tangent length " T' ", which we get on the basis of the distance of the points of intersection of the tangents and the tangent length of the unchanged curve with the following formula:

$$R' = \frac{T'}{\operatorname{tg} \frac{\alpha}{2}}$$

4.3. Sub Point Calculation in Plan

We name sub points those points, which are on the centreline of the road and from one another determined intervals as well as we want to know their layout data.

4.3.1. The Principle of the Calculation

To perform the calculations we have to know the coordinates of the points of traverse, the layout array, the stations array as well as the special points. The meaning of these are the following:

- layout array: the set of those data, in which we give to which station and from which side of the traverse we want to lay out the road centreline.
- stations array: the set of those data, in which we give the interval between the points of the road centreline, on the straight road sections and in the curves.
- special points: the set of those data, in which we give the name and the station of the points, which we want to lay out too.

We perform the calculations with the help of the program in two steps. First we determine the coordinates of the sub points "X, Y" considering the stations array and the special points, then the relative coordinates of the sub points " η , ξ " with the help of the layout array.

4.3.1.1. Used Procedures

We calculate the coordinates of the points "X, Y" depending on the characteristic of the points:

- a.) in straight section with the formula (3.6), in which the certain variables mean the following:

$$\begin{aligned} X &= X_i + \eta \cdot \cos\delta - \xi \cdot \sin\delta, \\ Y &= Y_i + \eta \cdot \sin\delta + \xi \cdot \cos\delta, \end{aligned} \tag{3.6}$$

- X_i, Y_i : the coordinates "X, Y" of the beginning of transition curve "BTC" which exits from the circular curve, respectively the end of curve "EC" in case of circular curve;
- η : the distance between the points of "BTC" or "EC" and the points, which we want to lay out;
- ξ : zero;
- δ : the azimuth of the tangent, which is between two curves.

b.) We get in the transition curve the relative coordinates "x, y" of the sub points referred to the tangent between two curves and calculated from the beginning of transition curve (BTC) with the formula (3.9), where:

$$x = l - \frac{l^5}{40p^4} + \frac{l^9}{3456p^8}, \quad y = \frac{l^3}{6p^2} - \frac{l^7}{336p^6} + \frac{l^{11}}{42240p^{10}}, \quad (3.9)$$

- p: the parameter of the entering and exiting transition curve;
- l: the distance between the beginning of transition curve and the points, which we want to lay out (difference between the stations).

After these we get the coordinates "X, Y" of the sub points similarly to the point a.) with the difference that we substitute instead of "η, ξ" in this case by the transition curve which is before the circular curve "x, y", by the transition curve which is after the circular curve "x, -y". By the exiting transition curve we take into consideration for value of "δ" with 180° increased value of azimuth of the tangent between point of "BTC" and the point of "BTC" or "BC" of the next curve. The reason for this is that because here the determination of the coordinates "X, Y" of the points occurs in the opposite direction with the direction of the placing station.

c.) On the circular curve with transition curves in case of sub points which are on the section of the circular curve the calculation of the values of abscissa of the circular curve's centre "X₀" and the circular curve's offset "ΔR" occurs with the formulas (3.8), (3.10) and (3.12). After these we get the relative coordinates "x, y" of the sub points similarly to the point b.) referred to the tangent, calculated from the point of "BTC" with the help of the following formulas:

$$(3.31)$$

$$\alpha_k = \frac{\Delta IH}{R} + \tau;$$

$$(3.32)$$

$$x = R \cdot \sin \alpha_k + X_0, \quad y = R \cdot (1 - \cos \alpha_k) + \Delta R.$$

where:

- ΔIH: the intervals between the points of circular curve and the end of transition curve (ETC);
- τ: centre angle belonging to the half-length of the transition curve;
- α_k: the sum of the centre angles belonging to the circular curve with length of "ΔIH" and the half-length of the transition curve.

Determination of the coordinates "X, Y" occurs according to the entering transition curve written at point b.).

- We get the relative coordinates " η, ξ " of the sub points referred to the side of traverse with the formula (3.7).

$$\begin{aligned}\eta &= (X - U) \cdot \cos\delta + (Y - V) \cdot \sin\delta, \\ \xi &= (Y - V) \cdot \cos\delta - (X - U) \cdot \sin\delta\end{aligned}\tag{3.7}$$

The certain variables mean the following:

- U, V: the coordinates of the beginning point of the traverse-side;
- X, Y: the coordinates of the sub points;
- δ : the azimuth of the traverse-side.

4.4. Calculations in the Profile

After determining the elevations of the points of the plan centreline we complete and display in AutoCAD the ground line of the profile. We perform the design of the vertical alignment similarly to the plan. Like horizontal alignment, vertical alignment has tangents (sustained grades) and curves. After the graphic design of the tangents and curves we make first the main point calculation and the necessary checkouts then the sub point computation.

4.4.1. Ground line of the Profile

To the finishing of the profile we need the elevations of the points of the plan centreline. To determination of these two possibilities arise:

- calculation of the elevations with the help of the terrain model,
- with levelling after laying out the points of the centreline.

We use the first variation at the pre-planning and the second one at the exact planning.

4.4.1.1. Principle of Calculation with the Help of the Terrain Model

Possibilities of producing of the terrain model:

- a) quadrangle network, generated on the basis of the measured terrain side slope at the points of grade line;
- b) triangulated irregular network (TIN), generated from the measured terrain points.

a) Quadrangle Network

The "i"-th and "i+1"-th points of the grade line and the points which are from these 10.00 metres in accordance with going direction to right perpendicularly to the traverse side (B_i , B_{i+1} , B'_i , B'_{i+1}) with their three coordinates can be determined with the help of the grade between the points of the grade line and the side slope measured at these points. The figure 4.4.1.1-1. shows the localization of the points.

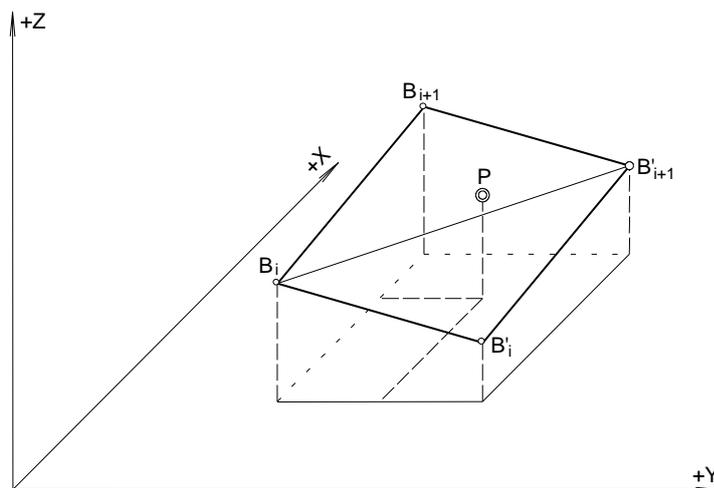


fig. 4.4.1.1-1.:
Spatial location of the terrain model

b) Triangulated Irregular Network

At the field not counting the measure of the points of the traverse and grade line we survey with total station the terrain approximately perpendicularly to the grade line. After analysing the data of the total station we can generate a triangulated irregular network with the help of the terrain points which have three coordinates (for example: maCADam). The advantage of TIN models opposite to the raster models are, that they can follow the directional changing of the space with smaller failure. To create a TIN model in general the Delaunay

triangulation has been used. In the TIN model those triangles satisfy the Delaunay condition, by which a circle passes through their three vertices (nodes of model) and within these points there is not any other node.

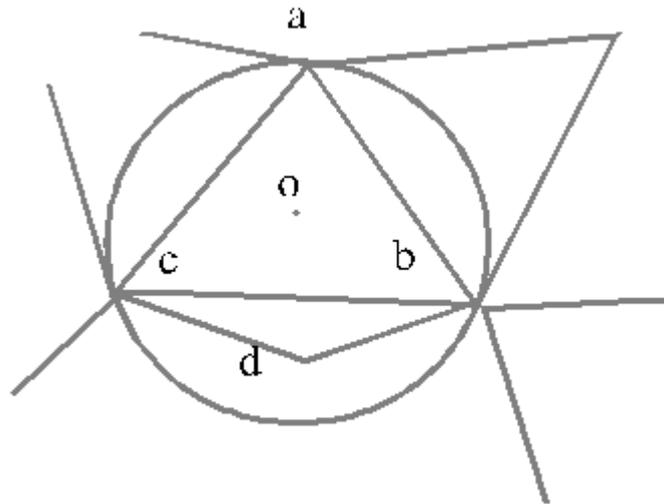


fig. 4.4.1.1-2.:

The triangle "a, b, c" in accordance with the Delaunay term.

4.4.1.1.1. Calculation of Elevations of the Centreline Points on the Terrain Model

The determination of the elevations of the centreline points within the spatial triangles occurs in the same way, after we have chosen the triangle within which the point with searched elevation is. In case of the quadrangle network the calculation of the elevation of the point, which has two coordinates ($P(X_p, Y_p)$) occurs in the following steps (fig. 4.4.1.1-1.):

- we have to state between which two grade line points the centreline point is,
- the calculation of the elevation with the help of the plain which is placed on the points " B_i, B_i', B_{i+1}' ".
- the calculation of the elevation with the help of the plain which is placed on the points " B_i, B_{i+1}, B_{i+1}' ".
- the elevation of the centreline point ($P(X_p, Y_p)$) presents itself by the average of the two previous elevations, which we weigh with the calculated distances between the centre line point and the horizontal projection of the " B_i, B_i' " and " B_{i+1}, B_{i+1}' " sections.

We use, for the computation, the following procedure and formulas:

- the using up of the points of the terrain model occurs with the help of the distances between the centre line point and the horizontal projection of the " B_{i+1}', B_{i+1} " sections which we calculate with the formula (3.7). The application of this we need the value " ξ " only. If the value with the

formula (3.7) is negative then we calculate the elevation of the centreline point among the "B_i, B_i', B_{i+1}, B_{i+1}' " points. If the previous value is positive then we compute the elevation of the centreline point among the "B_{i+1}, B_{i+1}', B_{i+2}, B_{i+2}' " points.

- parametral system of equations of the plane which is determined by the points "B_i(X₁, Y₁, Z₁), B_i'(X₂, Y₂, Z₂), B_{i+1}'(X₄, Y₄, Z₄)" (Moór, 1977):

$$\begin{aligned} X_P &= X_1 - U(X_1 - X_2) + V(X_4 - X_2) \\ Y_P &= Y_1 - U(Y_1 - Y_2) + V(Y_4 - Y_2), \\ Z_P &= Z_1 - U(Z_1 - Z_2) + V(Z_4 - Z_2), \end{aligned} \tag{3.33}$$

from which after expressing "U" and "V" then substituting into the third equation, the first elevation "Z_{P1}" of the centreline point "P(X_P, Y_P, Z_P)":

$$Z_{P1} = \frac{-M_1(X_P - X_1) - M_2(Y_P - Y_1) + M_3 Z_1}{M_3}, \tag{3.34}$$

where the values "M₁, M₂, M₃ " are obtainable with the following formulas:

$$\begin{aligned} M_1 &= (Y_1 - Y_2)(Z_4 - Z_2) - (Y_4 - Y_2)(Z_1 - Z_2) , \\ M_2 &= (X_4 - X_2)(Z_1 - Z_2) - (X_1 - X_2)(Z_4 - Z_2), \\ M_3 &= (X_1 - X_2)(Y_4 - Y_2) - (Y_1 - Y_2)(X_4 - X_2). \end{aligned} \tag{3.35}$$

- the wanted second elevation "Z_{P2}" can be calculated also with the formula (3.34) on the plane, which is determined by the points "B_i(X₁, Y₁, Z₁), B_{i+1}(X₃, Y₃, Z₃), B_{i+1}'(X₄, Y₄, Z₄)". We substitute into the formula (3.34) instead of "M₁, M₂, M₃" the values "D₁, D₂, D₃" according to the following:

$$\begin{aligned} D_1 &= (Y_1 - Y_3)(Z_4 - Z_3) - (Y_4 - Y_3)(Z_1 - Z_3) , \\ D_2 &= (X_4 - X_3)(Z_1 - Z_3) - (X_1 - X_3)(Z_4 - Z_3), \\ D_3 &= (X_1 - X_3)(Y_4 - Y_3) - (Y_1 - Y_3)(X_4 - X_3). \end{aligned} \tag{3.36}$$

- the elevation "Z_P " of the point "P" on the basis of the terrain model:

$$Z_P = \frac{T_2 \cdot Z_{P1} + T_1 \cdot Z_{P2}}{T_1 + T_2}, \tag{3.37}$$

where:

- T_1 : distance between the point "P(X_p , Y_p)" and the horizontal projection of the " B_i , B_i ' " section;
- T_2 : distance between the point "P(X_p , Y_p)" and the horizontal projection of the " B_{i+1} , B_{i+1} ' " section.

4.4.2. Calculation of Main Points in the Profile

This calculation step serves the determination of the station values and the elevations of the vertical points of intersection as well as the beginning of vertical curves "BVC" and the end of the vertical curves "EVC" of the graphically designed vertical alignment. We name the vertical points of intersection as well as the beginning of vertical curves "BVC" and the end of the vertical curves "EVC" main points of the vertical alignment.

4.4.2.1. The Principle of the Calculation

The graphically designed vertical tangents (sustained grades) bringing to intersect we get a vertical traverse. With the help of this, the calculation of the main point occurs according to the following:

- a.) If the grade difference of the two consecutive vertical tangents is more than 1.00%, we design a vertical curve. The computation occurs with the grade changing method in the following steps (fig. 4.4.2.1.1-1.):
 - specifying the grades of the vertical tangents as well as the station values and elevations of their one-one point, the station value end the elevation of the intersection point of two consecutive vertical tangents can be determined;
 - specifying the grade difference between the two tangents and on the basis of the projection length of the vertical curve the percentage value of the grade changing as well as the length of the short sustained grades, the radius and length of the vertical curve further the station value and the elevation of the beginning "BVC" and the end "EVC" of covering polyline.
- b.) If the grade difference of two consecutive vertical tangents is smaller than 1.00%, it is sufficient just to calculate the point of intersection, which we can put to a centreline point. At this time the elevation of the point of intersection must be computed.

4.4.2.1.1. Used Procedures

The determination of the main points occurs with the help of following formulas (fig. 4.4.2.1.1-1.):

- the station value " x_s " and the elevation " z_s " of the point of intersection of two consecutive vertical tangents can be calculated with the following formulas:

$$x_s = \frac{z_2 - z_1 + x_1 g_1 - x_2 g_2}{g_1 - g_2}, \quad z_s = z_1 + (x_s - x_1) g_1, \quad (3.38)$$

in which:

- x_1, x_2 and z_1, z_2 : the station values and elevations of the one-one point of two consecutive tangents;
- g_1, g_2 : the upgrade or slope value of the two tangents expressing in ratio ($g = g\% / 100$), where the sign is by upgrade "+" by slope "-".

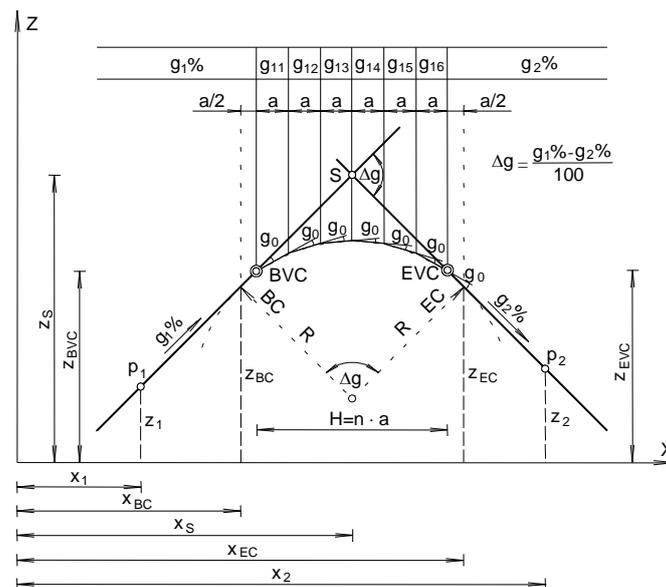


fig. 4.4.2.1.1-1.:
Vertical curve with the grade changing method

- the radius of the vertical curve "R":

$$R = \frac{100}{g_0\%} \cdot a, \quad (3.39)$$

where:

- a: the lengths of the side of the covering polyline of the vertical curve;
- $g_0\%$: the value of the grade changing, by which the sign is by crest curve positive by sag curve negative. At the same time this determines the sign of the radius "R" too.
- on the basis of the marks used in the (3.38) and (3.39) formulas, the projection of half of the vertical curve otherwise the half arc length with good approximation "T":

$$T = \left| \frac{R(g_1 - g_2)}{2} \right|. \quad (3.40)$$

- using up the station value of the point of intersection " x_S " computed with the formula (3.38), the station value of the beginning (BC) and the end (EC) of vertical curve " x_{BC}, x_{EC} ":

$$x_{BC} = x_S - T, \quad x_{EC} = x_S + T. \quad (3.41)$$

- the number of sides of the "i"-th vertical curve substituting covering polyline " n_i ", the length of polyline " H_i " as well as the station value and elevation of the beginning and the end of polyline " $x_{BVC(i)}, x_{EVC(i)}, z_{BVC(i)}, z_{EVC(i)}$ ":

$$n_i = \left| \frac{(g_{1(i)} - g_{2(i)}) - g_{0(i)}}{g_{0(i)}} \right|; \quad H_i = n_i \cdot a_i; \quad (3.42)$$

$$x_{BVC(i)} = x_{BC(i)} + \frac{a_i}{2}; \quad x_{EVC(i)} = x_{EC(i)} - \frac{a_i}{2}; \quad (3.43)$$

$$z_{BVC(i)} = z_{EVC(i-1)} + (x_{BVC(i)} - x_{EVC(i-1)})g_{1(i)}; \quad (3.44)$$

$$z_{EVC(i)} = z_{BVC(i)} + \sum_{k=1}^{n_i} gl_k \cdot a_i, \quad \text{where } gl_k = gl_{k-1} - g_{0(i)}, \quad gl_0 = g_{1(i)} \quad \text{and}$$

the value of "k" goes from 1 to " n_i ".

The explanation of the used marks in the formulas (3.42), (3.43) and (3.44) are the following:

- $g_{1(i)}$, $g_{2(i)}$: the slope or upgrade value of the two tangents given in ratio, which are tangential to the "i"-th vertical curve;
- $x_{EVC(i-1)}$, $z_{EVC(i-1)}$: the station and the elevation of the end of prior vertical curve, before "i"-th curve;
- $x_{BVC(i)}$, $x_{EVC(i)}$: the station values of the beginning and the end of the "i"-th vertical curve;
- g_k : the slope or upgrade value of the "k"-th side of the polyline specified in ratio;
- a_i : the length of one side of the covering polyline;
- $g_{0(i)}$: the value of the grade changing specified in ratio (its sign is by the crest curve "+" by sag curve "-").

4.4.2.1.2. Input Data to the Grade Changing Method

The input occurs parallel with the grade changing method in accordance with the following:

- we draw the appropriate vertical curve to the ground line
- we determine the horizontal distance between the points of contact "BC, EC" ($x_{BC} - x_{EC}$);
- we choose the value " g_0 " in such a way that the grade difference between two tangents (Δg) dividing with " g_0 " we should get integer value " n_0 ";
- we calculate the same length of the sides of the substitutive covering polyline "a" with the formula " $(x_{BC} - x_{EC}) / n_0$ ".

The " g_0 " is the value of the grade changing with hundredth percent accuracy. The sign of " g_0 " is by the crest curve positive by sag curve negative. We give the length "a" with doubles centimetre accuracy.

4.4.3. Calculation of Sub Points in the Profile

This calculative step serves the determination of the elevations of the road centreline points according to the designed vertical alignment.

4.4.3.1. The Principle of the Calculation

At the certain station points the calculation of the elevation of the designed road centreline according to the vertical alignment occurs in two ways:

- **on the section of the vertical tangent, which is between two main points**, from the previous point, whose elevation is already determined, on the basis of the difference of station and the upgrade or slope value;
- **by the vertical curves**
 - by grade changing method on the basis of the same length of the sides and with the grade changing (g_0) modified upgrade or slope value of the substitutive covering polyline;
 - with the help of the parabola formula.

In the course of determination of the sub points we compute the main points too, ensuring with this, the possibility of the checkout.

4.4.3.1.1. Used Procedures

The compute of sub points occurs with the help of the following relations:

- on the section between the main points of the vertical tangent at the "n"-th station the elevation of the vertical road centreline. " Z_n ":

$$Z_n = Z_{n-1} + (s_n - s_{n-1}) \cdot \frac{g_i \%}{100}, \quad (3.45)$$

where:

- $g_i\%$ is the upgrade (+) or slope (-) value of the "i"-th tangent given in ratio;
- s_{n-1}, s_n : the station value of the "n-1"-th and "n"-th centreline point;
- Z_{n-1} : the elevation of the vertical road centreline in the previous point.

- **in vertical curve:**

at the grade changing method:

- in the "k"-th station " $s_{PC(k)}$ " the elevation of the vertical road centreline " $Z_{PC(k)}$ " which is after the "i"-th vertical tangent ($g_i\%$) on the "n_j"-th side of the vertical curve substituting covering polyline:

$$g_{l_j} \% = g_{l_{j-1}} \% - g_0 \% , \text{ (if } j = 1 \text{ then } g_{l_0} \% = g_i \% \text{);} \quad (3.46)$$

$$Z_{PC(j)} = Z_{PC(j-1)} + \frac{g_{l_j} \%}{100} \cdot a, \text{ (if } j = 1 \text{ then } Z_{PC(0)} = Z_{BVC} \text{);}$$

$$S_{PC(j)} = x_{BVC} + n_j \cdot a;$$

$$z_{PC(k)} = Z_{PC(j)} + (s_{PC(k)} - s_{PC(j)}) \cdot \frac{gl_j \%}{100}.$$

The explanation of the marks is the following:

- $s_{PC(j)}$, $Z_{PC(j)}$: the station value and the elevation of the point of intersection which means the endpoint of the "j"-th side of the covering polyline.
- $Z_{PC(j-1)}$: the elevation of the end of the "j-1"-th side of polyline;
- x_{BVC} , Z_{BVC} : the station value and the elevation of the beginning of the vertical curve (first point of the polyline)
- $gl_{j-1}\%$, $gl_j\%$: the percentage value of the upgrade (+) or slope (-) value of the "j-1"-th and "j"-th side of polyline;
- $g_0\%$: the value of the grade changing (by the crest curve "+" by sag curve "-")
- a : the length of one side of the covering polyline.

with parabola formula:

- we determine the station value and the elevation of the beginning and the end of the vertical curve similarly to the grade changing method with the difference that we do not subtract the $a/2$ value from the length of the tangent. We get the elevation of whichever point on the curve in such a way that, at first, we calculate the elevation of the point with the wanted station value on the tangent, then calculating with

$$y = \frac{x^2}{2R}$$

- the parabola formula and subtracting or adding the elevation difference "y" from or to the previous elevation we get the elevation of the vertical road centreline on the curve.

4.5. Calculation of the Widening and Superelevation Shaping

The purpose of the calculation is the determination of the station value of the beginning of shaping "BS" and the end of shaping "ES" as well as the measure of the widening and superelevation connecting the curves in plan. We need this in the design-operation of the calculation of the earthwork volume, in order to determine the breakpoints further the area of the cut and fill of the cross-sections.

4.5.1. Principle of the Calculation

We perform the calculation of the widening and superelevation shaping of the curves, which are determined in the plan after specifying the width and in the straight road section applied side slope of the surface, on the basis of the following principles and order:

- the determination of the maximum value of the widening and the superelevation (one-sided slope in %) occurs with the help of tables, by widening as a function of the radius and central angle, by the superelevation on the basis of the radius only;
- we solve the shaping of the widening by circular curves along fore-curve with double radius, by circular curves with spiral transition curves along the transition curve in a linear way;
- we perform the shaping of the superelevation along the shaping of the widening, with rotating the half of surface around the surface level, which is at the centreline;
- if the relative grade of the outside edge of surface surpasses in case of the first-class road the 1.0%, in case of second-class road the 1.5%, then we calculate the length of the shaping of the superelevation with the help of these limits. This length in case of the circular curve with spiral transition curves we connect to the point of the end of transition curve, by the circular curve we locate the quarter along the curve three-quarters in the straight road section.
- if the radius longer then 250.00 m we ignore the widening. We calculate the necessary length of the superelevation shaping in this case from the 1.0% relative grade of the outside edge of surface. We place the half of this in the curve, another half in the straight section. In case of second-class road in so far as in this way get length has not room instead of the preceding 1.0% we compute with the limit of 1.5%.

In accordance with the going direction we calculate the exit from the first curve and the entry into second curve shaping section in one course with the consideration of the following:

- a.) By reversed curves if one curve is circular curve we have to determine for the shaping necessary length of the straight section. By inflexion curves if at the straight section the road surface has a two-directional slope (camber) we do not create saddle transition, but we place the "0%" slope at the inflexion point and the road surface is made into superelevation, which requires the second curve.
- b.) By compound curves if the straight section between two curves is shorter than with 10 m increased length of shaping (broken back), then after the exit, from the first curve, we do not put back the two half-

surface in accordance with the straight section slope. If the widening or superelevation of the first curve is the greater, then we create the required value in the second curve along the length of shaping belonging to the first curve. If the widening or superelevation of the second curve is the greater, then we keep the value of the first curve to the beginning point of shaping of the second curve and connect the transition section to this curve.

- c.) If between the two curves the straight section is longer than the above mentioned, we calculate the shaping of the two curves independently from one another.

4.5.1.1. Used Procedures

To the calculation of the shaping of the superelevation and widening we have to determine the following values:

- superelevation of the outside half-surface;
- minimum length of the shaping;
- length of shaping of the widening in case of the circular curve;
- values of the superelevation and widening in the shaping section.

Superelevation of the Outside Half-surface (se_{\max}):

- a.) If at the straight section the road surface has a two-directional slope (camber) or the applied one-sided slope is in contradiction with the direction of the required side slope in the curve:

$$se_{\max} = S \cdot \frac{(q\% + d\%)}{200}. \quad (3.47)$$

- b.) If at the straight section applied one-sided slope is equal with direction of the required side slope in the curve:

$$se_{\max} = S \cdot \frac{(q\% - d\%)}{200}. \quad (3.48)$$

Minimum Length of Shaping (D_m):

- a.) If at the straight section is a two-directional slope (camber) or applied one-sided slope is in contradiction with the direction of the required side slope in the curve:

(3.49)

$$D_m = S \cdot \frac{(q\% + d\%)}{2g_r \%}$$

b.) If at the straight section applied one-sided slope is equal with direction of the required side slope in the curve:

(3.50)

$$D_m = S \cdot \frac{(q\% - d\%)}{2g_r \%}$$

The used marks in the above formulas are the following:

- S: the road surface width;
- q%: the required side slope in the curve;
- d%: at the straight section applied one-sided slope or two-directional slope (slope of roof section);
- g_r %: the relative grade of the outside edge of surface.

The Length of Projection of the Applied Fore-Curve with Double Radius by the Shaping of Widening of the Circular Curve (D)

(3.51)

$$R_b = R - (S/2 + \Delta S), \quad \Delta R_b = \Delta S,$$

$$D = 2 \cdot \sqrt{2R_b \cdot \Delta R_b - \Delta R_b^2},$$

where:

- R_b: the radius of the widened inside edge of surface;
- ΔR_b: the circular curve's offset of the fore-curve with double radius;
- ΔS: the maximum value of the required widening in the curve.

We determine the widening "ΔS_i" and superelevation "se_i" values at the centreline points which are on the shaping section as a function of the distance calculated from the beginning point of shaping linear method.

4.6. Calculation of Earthwork Volumes

To the calculation of the areas of the cut and fill of the cross-sections we have to know the data of the cross-section template for straight road-sections. At the exact planning, besides, the straightened line of the bottom of ditch must be designed in the profile. After the graphical straightening the points of the bottom of ditch, the elevation of the bottom of ditch of the certain station can be

calculated on the basis of the elevation of the beginning- and endpoint of the sustained grade of the bottom of ditch.

On the basis of the results which we get with the procedures introduced so far, the cross-sections and the calculation of earthwork volumes can be finished, which makes the comparison of the design variations possible as well as at the chosen variation shows that sections which we modify and so we get a more advantageous solution. The accuracy of the calculation depends on the input data. We produce the ground lines of the cross-sections in the phase of pre-planning with the help of the terrain model. At the exact planning we get this on the field at the laid out centreline points with survey of ground perpendicular to the centreline (for example stave with spirit level). If we have measured the breakpoints of ground with the grade line together with the total station in the width necessary to design (20-20 m), then the terrain model produced with the help of this insures proper accuracy to the calculation of earthwork volumes.

4.6.1. Principle of the Calculation

We compute the approximate areas of the cuts and fills of the cross-sections belonging to the centreline points (stations) on the basis of the ground lines of the cross-sections and concerning data to the subgrade – subgrade or equalizing subgrade level – of the cross-section template for straight road sections, straightened bottom of ditch levels as well as in previously established results. Thereafter we determine the earth volumes of the cuts and fills with the help of the cross-section areas and the distances between the stations. If we summarize and plot like profile form sign-properly the deficit and the surplus of earth volumes, which arise between the station segment (cross-sections), then we get the mass haul diagram (fig. 4.6.1-1.). On the basis of this modifying the vertical alignment more favourable distribution of earth volume can be reached. The steps of the performance are the following:

- a) The computation of the area is realizable if we give the modified ditch depth (at exact planning the elevation of straightened bottom of ditch) the angles of the slopes and the thickness of the humus layer. At first we disassemble the area between the road prism and humus layer from the centreline to the left then to the right on trapezoids and triangles, with which we determine the elevation of their vertexes as well as their distance from the centreline. Thereafter the area of the geometric shapes can be calculated and summarized, which mean the areas of the cuts and fills.

By another possibility in accordance with the principle of chord planimeter we disassemble the cross-sections with parallel lines which

are small distance from one another on trapezoids, then we summarize their areas.

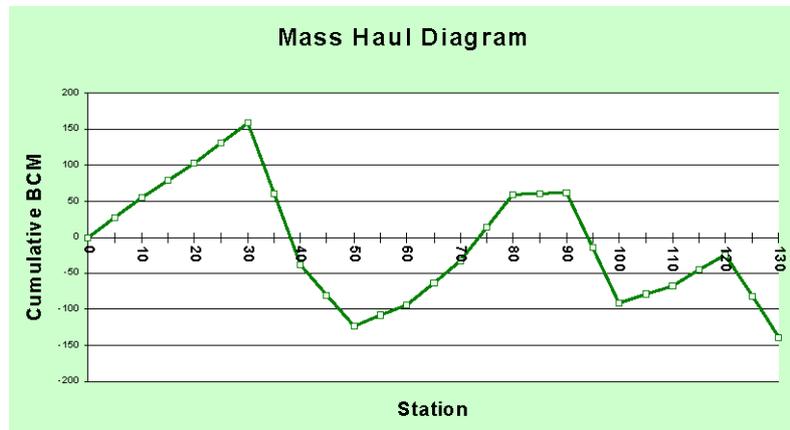


fig. 4.6.1-1.:
Mass haul diagram

- b) After increasing the areas of fill with 10%, if we multiply the sum of neighbouring areas with the half of distance between the cross-sections, we get the approximate earth volume. In case of transitional body – a through cut cross-section follows a through fill cross-section or inversely – the earth volume consists of the part of before and after the transitional point.

4.6.1.1. Used Procedures

The calculation of earth volume occurs with the following formulas:
- from the centreline calculated distance and the elevation of intersection point of two straight lines (d, z):

$$d = \frac{b_1 - b_2}{m_2 - m_1}, \quad z = m_1 \cdot d + b_1, \quad (3.52)$$

in which:

- b_1, b_2 : elevations of the intersection points of two straight lines and the "Z" axis;
- m_1, m_2 : steepness of the two straight lines.
- area of such a trapezoid, which from its two parallel sides one is the "Z" axis (A_{tr}):

$$(3.53)$$

$$A_{tr} = \left[\frac{|z_1 - z_2| + |z_3 - z_4|}{2} \right] \cdot d_3 ,$$

where:

- z_1, z_2, z_3, z_4 : the elevations of the vertexes of the trapezoid;
- d_3 : distance between the parallel sides of trapezoid.

- area of the triangle (A_{ta}):

$$A_{ta} = \frac{|d_1(z_2 - z_3) + d_2(z_3 - z_1) + d_3(z_1 - z_2)|}{2}, \quad (3.54)$$

by which:

- z_1, z_2, z_3 : elevations of the vertexes of the triangle;
- d_1, d_2, d_3 : distances of the vertexes from the "Z" axis.

- if the " A_f " means the area of fill, " A_c " means the area of cut and "SV" means the station value, then the fill and cut earth volume (V_f, V_c):

$$V_f = \frac{A_{f(i)} + A_{f(i+1)}}{2} \cdot (SV_{i+1} - SV_i), V_c = \frac{A_{c(i)} + A_{c(i+1)}}{2} \cdot (SV_{i+1} - SV_i). \quad (3.55)$$

- the volume of the transitional body between " $SV_{(i)}$ " and " $SV_{(i+1)}$ " stations (V_{tf}, V_{tc}):

$$D = SV_{i+1} - SV_i, D_f = \frac{D}{M_f + M_c} \cdot M_f, D_c = D - D_f, \quad (3.56)$$

$$V_{tf} = \frac{A_t}{2} \cdot D_t, V_{tc} = \frac{A_c}{2} \cdot D_c.$$

The explanation of the applied marks is the following:

- D : distance between the stations;
- M_f : the measure of fill at the "Z" axis;
- M_c : the measure of cut at the "Z" axis;
- D_f : distance between the segment of fill and the transitional point;

- D_c : distance between the segment of cut and the transitional point;
- A_f : area of fill;
- A_c : area of cut.